Equity Volatility Estimate

Field Test Runs

Current Volatility Value

AIRG Equity Model has a Stochastic Volatility component

Some Insurers need to have it for Field Test

However, Conning's GEMS Model has a different Structure

$$\frac{dS(t)}{S(t)} = [(r(t) - D(t)) + \mu_0 + \mu_1 V(t) - \lambda m V(t)]dt + \sqrt{V(t)}dW_1(t) + \gamma dN(t)$$

$$dV(t) = \big(\alpha - \beta V(t)\big)dt + \sigma\sqrt{V(t)}dW_2(t)$$

$$dD(t) = \kappa\big(\alpha_D + \theta z(t) - D(t)\big)dt + \sigma_D\sqrt{D(t)}dW_3(t) + D(t)\gamma_D dN(t)$$

Need a mapping for Field Test

AIRG Volatility Value

Given Participants Usage, the focus is next Simulation Month's Impact

In the AIRG, the Volatility Value has two Roles in this Context

1. It affects expected returns
2. It defines volatility

Based on Participant Usage, Conning is focusing on the Second Component

GEMS Comparison

GEMS Equity Model has 5 Sources of Volatility that Need to be Considered

1. Price Volatility
2. Volatility of the Volatility
3. Dividend Volatility
4. Jump Process
5. Overnight Treasury Yield

We can ignore #2 since there is a Comparable, Separate Parameter in the AIRG

Need to Consider the Others

GEMS Comparison - Dividend

AIRG Model only Projects Total Returns

This will Allow us to Ignore the Dividend Yield

- In GEMS, Total Return is, roughly, Price Return + Dividend Yield / 12
- However, Price Return's Drift is reduced by current Dividend Yield

$$\frac{dS(t)}{S(t)} = [(r(t) - D(t)) + \mu_0 + \mu_1 V(t) - \lambda m V(t)]dt + \sqrt{V(t)}dW_1(t) + \gamma dN(t)$$

- Means Dividend Yield has very little impact on Total Return distribution

GEMS Comparison – Overnight Yield

GEMS Total Return is Linked to Overnight Treasury Yield

$$\frac{dS(t)}{S(t)} = [(r(t) - D(t)) + \mu_0 + \mu_1 V(t) - \lambda m V(t)]dt + \sqrt{V(t)}dW_1(t) + \gamma dN(t)$$

Means Expected Volatility is Linked to Current Yields

- Specifically, higher yields => higher equity volatility

GEMS Comparison – Overnight Yield (cont.)

However, Impact is Minimal

- Only looking at next month
- Yield changes over that period are only a fraction of equity volatility

Initial Value	Run #1a	Run #1c	Difference
0.005	0.106736246	0.106703996	0.000032250
0.01	0.135134360	0.135108888	0.000025471
0.015	0.158524333	0.158502621	0.000021712
0.02	0.178881618	0.178862377	0.000019241

Source: Conning GEMS Economic Scenario Generator

So, Conning will Ignore this Source for this Conversion

GEMS Comparison – Remaining Drivers

That Leaves two Items we Need to Consider

- Price Volatility
- Jump Process

$$\frac{dS(t)}{S(t)} = [(r(t) - D(t)) + \mu_0 + \mu_1 V(t) - \lambda m V(t)]dt + \sqrt{V(t)}\, dW_1(t) + \gamma dN(t)$$

Given Participants' Usage, Conning is Focusing on Expected Volatility

- The Jump Process (i.e. N(t)) is Poisson with mean of $\lambda V(t)$
- Means we can estimate the desired value based solely on the current $V(t)$
- Mapping WILL depend on Equity parameters, especially λ

GEMS Comparison – Native Equity Mapping

Based on the AIRG's Construction, this Parameter align with

- Annualized Sigma
- Log of next month's Total Return

Conning does NOT have a tool which Calculates this exact value

- All of Conning's tools are forward looking
- So, closest estimate available is:

1. Standard Deviation
2. Continuously Compounded Total Return
3. At end of the next simulated month

- Will use this value as the basis of the conversion formula

GEMS Comparison – Native Equity Mapping (cont.)

Specifically, Conning created a Table of this value with Different Initial Volatilies

- For the 3 relevant funds, the Run #1a table looks like this:

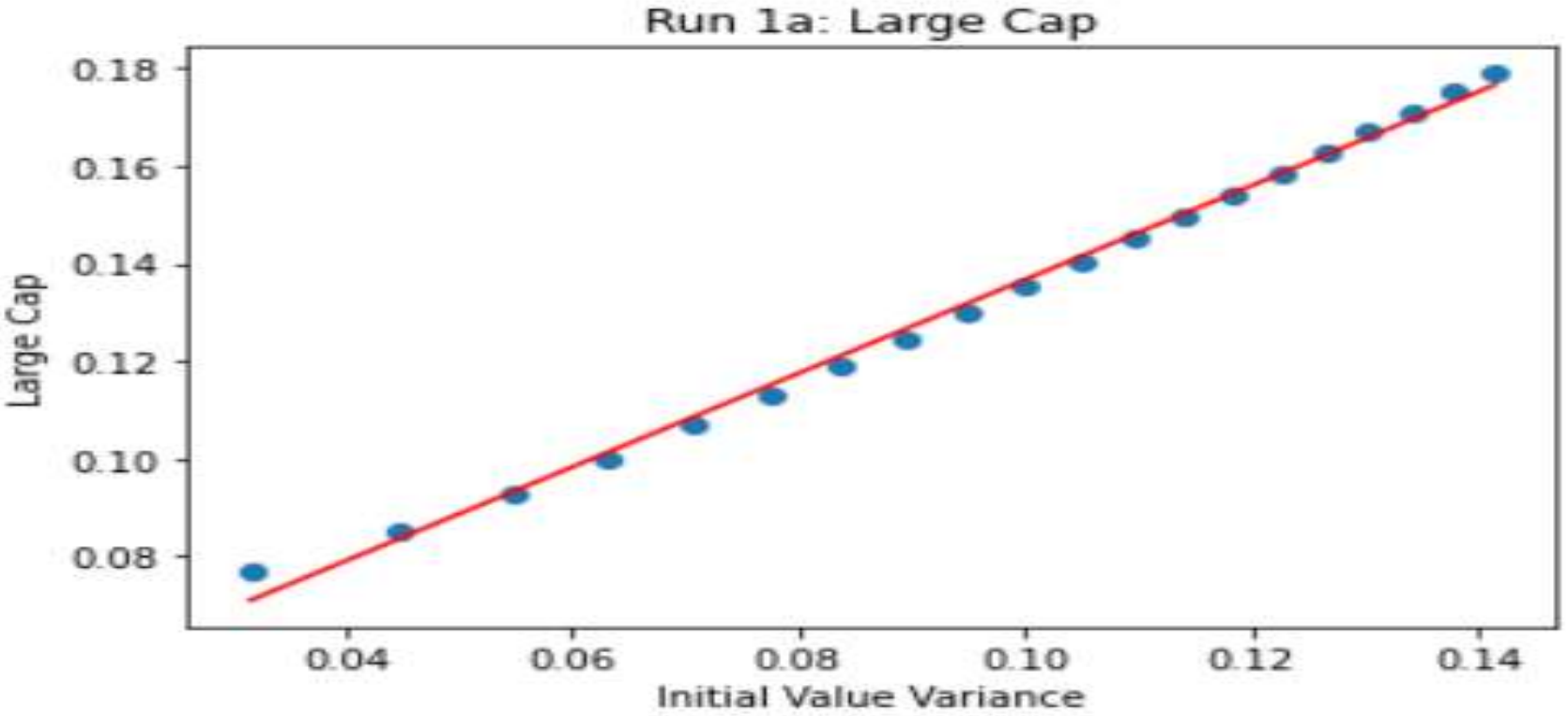
Initial Value	Large Cap	Small Cap	Aggressive US Equity
0.001	0.076796425	0.077968449	0.091895230
0.002	0.085272649	0.087640179	0.099725439
0.003	0.092979345	0.096345850	0.106984079
0.004	0.100094417	0.104327584	0.113780592
0.005	0.106736246	0.111740623	0.120193398
0.006	0.112988318	0.118691570	0.126280968
0.007	0.118912126	0.125257378	0.132088276
0.008	0.124554516	0.131495751	0.137650800
0.009	0.129952150	0.137451281	0.142997107
0.01	0.135134360	0.143159270	0.148150608
0.011	0.140125048	0.148648237	0.153130769
0.012	0.144943999	0.153941613	0.157953988
0.013	0.149607809	0.159058927	0.162634228
0.014	0.154130563	0.164016658	0.167183498
0.015	0.158524333	0.168828866	0.171612214
0.016	0.162799564	0.173507660	0.175929480
0.017	0.166965361	0.178063556	0.180143309
0.018	0.171029721	0.182505759	0.184260798
0.019	0.174999713	0.186842377	0.188288268
0.02	0.178881618	0.191080600	0.192231375

Source: Conning GEMS Economic Scenario Generator

GEMS Comparison – Native Equity Mapping (cont.)

Since GEMS Value is a Volatility, Conning did a Regression between its Square Root and the Target Value

- Resulting fit is very tight



Source: Conning

GEMS Comparison – Native Equity Mapping (cont.)

Final Result is these Mappings:

	GEMS Equity Calibration		Alternative Equity Calibration	
Index	Slope	Intercept	Slope	Intercept
Large Cap	1.0335	0.0402	0.9601	0.0407
Small Cap	1.0940	0.0394	1.0616	0.0385
Aggressive US Equity	0.9717	0.0557	0.9473	0.0553

Source: Conning

GEMS Comparison – Non-Native Equity Mapping

The Approach can't be Applied to Non-Native Equity Indices

Those are Based on Regression Models

- Results in a completely different model structure

Means we need to Develop Alternative Method for Aggressive Equity Fund

For that, Conning went back to the Underlying Formulas

- **Aggressive Equity** = 2/3 * **Aggressive Foreign Equity** + 1/3 * **Aggressive US Equity**
- **Aggressive Foreign Equity** = 1.048 * **US Large Cap** + 0.282 * **International Diversified Equity** + **Error Term**$_1$
- **International Diversified Equity** = 0.864 * **US Large Cap** + **Error Term**$_2$

GEMS Comparison – Non-Native Equity Mapping (cont.)

Rearranging and Combing Terms, we get the following:

- **Aggressive Equity** = 0.8611 * **US Large Cap** + 1/3 * **Aggressive US Equity** + 2/3 * **Error Term**$_1$ + 0.188 * **Error Term**$_2$

This Yields the following Equation for the Aggressive Equity's Variance:

- Var[**Aggressive Equity**] = 0.7415 * Var[**US Large Cap**] + 0.1111 * Var[**Aggressive US Equity**] + 0.4880 * σ[**US Large Cap**] * σ[**Aggressive US Equity**] + 0.1062 ^ 2 + 0.0201 ^ 2

Similar Process for International Equity Produces:

- Var[**International Diversified Equity**] = 0.7465 * Var[**US Large Cap**] + 0.0123

Conning will output the Square Root of these Formula for these Equities

Field Study Data

Each Run will have Four Separate Files Added

- One each for Large Cap, Small Cap, International Diveresified and Aggressive Equity
- Based on this methodology
- 10,000 Scenarios as Rows, 1,201 Months as columns

Data will NOT be added to any of the Subsets

- Designed to avoid confusion
- To get the subset data, link the scenario number in the first column to the value if the full 10,000 scenario set